

## 12 Quadratic Functions

### 12-1 Quadratic Equations with Perfect Squares

**Objective:** To solve quadratic equations involving perfect squares.

#### Vocabulary

**Perfect square** An expression such as  $x^2$ ,  $(x - 1)^2$ , or  $(2x + 5)^2$ .

**Roots of  $x^2 = k$**  An equation in the form “perfect square =  $k$ ” ( $k \geq 0$ ) can be solved by the method shown in Examples 1 and 2.

If  $k > 0$ , then  $x^2 = k$  has **2** real roots:  $x = \pm\sqrt{k}$ .

If  $k = 0$ , then  $x^2 = k$  has **1** real root:  $x = 0$ .

If  $k < 0$ , then  $x^2 = k$  has **no** real roots.

**Example 1** Solve:    a.  $m^2 = 36$     b.  $3r^2 = 48$     c.  $x^2 - 11 = 0$     d.  $m^2 = -25$

**Solution**    a.  $m^2 = 36$   
 $m = \pm\sqrt{36}$   
 $m = \pm 6$

The solution set is  $\{6, -6\}$ .

b.  $3r^2 = 48$   
 $r^2 = 16$   
 $r = \pm\sqrt{16} = \pm 4$

The solution set is  $\{4, -4\}$ .

c.  $x^2 - 11 = 0$   
 $x^2 = 11$   
 $x = \pm\sqrt{11}$

The solution set is  $\{\sqrt{11}, -\sqrt{11}\}$ .

d.  $m^2 = -25$   
 Since the square of any real number is always a nonnegative number, there is *no real solution*.

**Solve. Express irrational solutions in simplest radical form. If the equation has no solution, write *No solution*.**

1.  $x^2 = 49$

2.  $2x^2 = 18$

3.  $x^2 = \frac{25}{36}$

4.  $a^2 = -16$

5.  $2x^2 = 128$

6.  $5x^2 = 125$

7.  $9x^2 = 81$

8.  $x^2 - 81 = 0$

9.  $x^2 + 25 = 0$

10.  $m^2 - 10 = 0$

11.  $0 = 6x^2 - 24$

12.  $0 = 3m^2 - 75$

**Example 2** Solve  $(x + 3)^2 = 49$

**Solution**     $(x + 3)^2 = 49$   
 $x + 3 = \pm\sqrt{49}$   
 $x = -3 \pm 7$   
 $x = 4$  or  $x = -10$

**Check:**     $(4 + 3)^2 \stackrel{?}{=} 49$      $(-10 + 3)^2 \stackrel{?}{=} 49$   
 $7^2 \stackrel{?}{=} 49$      $(-7)^2 \stackrel{?}{=} 49$   
 $49 = 49 \checkmark$      $49 = 49 \checkmark$

The solution set is  $\{4, -10\}$ .

Note that  $(x + 3)^2$  is a perfect square.  
 Find the *positive or negative* square root of each side.  
 Subtract 3 from each side.

**12-1 Quadratic Equations with Perfect Squares** (continued)

Solve. Express irrational solutions in simplest radical form. If the equation has no solution, write *No solution*.

13.  $(x - 3)^2 = 0$

14.  $(z - 1)^2 = 16$

15.  $(r - 5)^2 = 100$

16.  $(x - 1)^2 = 25$

17.  $(2x + 9)^2 = 225$

18.  $(t - 4)^2 = 9$

**Example 3** Solve: a.  $3(x - 2)^2 = 21$

b.  $y^2 + 10y + 25 = 36$

**Solution**

a.  $3(x - 2)^2 = 21$

$(x - 2)^2 = 7$

$x - 2 = \pm\sqrt{7}$

$x = 2 \pm \sqrt{7}$

$x = 2 + \sqrt{7}$  or  $x = 2 - \sqrt{7}$

The check is left to you.

The solution set is

$\{2 + \sqrt{7}, 2 - \sqrt{7}\}$ .

b.  $y^2 + 10y + 25 = 36$

$(y + 5)^2 = 36$

$y + 5 = \pm\sqrt{36}$

$y + 5 = \pm 6$

$y = -5 \pm 6$

$y = 1$  or  $y = -11$

The check is left to you.

The solution set is  $\{1, -11\}$ .

*Note:* Example 3(b) could also have been solved by factoring.

Solve. Express irrational solutions in simplest radical form. If the equation has no solution, write *No solution*.

19.  $9m^2 - 1 = 35$

20.  $27 = 2r^2 - 5$

21.  $3x^2 - 9 = 33$

22.  $64 = 2t^2 - 8$

23.  $2n^2 + 6 = 38$

24.  $7x^2 + 1 = 64$

25.  $3(m - 2)^2 = 15$

26.  $400 = 4(z - 2)^2$

27.  $2(x - 5)^2 = 98$

28.  $25 = (2x + 1)^2$

29.  $5(m - 3)^2 = 80$

30.  $6(z + 5)^2 = 216$

31.  $3(x - 1)^2 = -24$

32.  $(3x - 1)^2 + 12 = 4$

33.  $6(x + 5)^2 = 24$

34.  $7(x + 2)^2 = 112$

35.  $(x - 2)^2 - 1 = 35$

36.  $2(3n - 1)^2 = 8$

37.  $3(2x - 1)^2 = 27$

38.  $2(x + 3)^2 - 4 = 68$

39.  $5(x - 1)^2 + 3 = 23$

40.  $x^2 - 2x + 1 = 9$

41.  $x^2 + 18x + 81 = 98$

42.  $x^2 - 12x + 36 = 64$

43.  $x^2 - 4x + 4 = 16$

44.  $x^2 + 10x + 25 = 81$

45.  $n^2 - 8n + 16 = 36$

**Mixed Review Exercises**

Express each square as a trinomial.

1.  $(x - 8)^2$

2.  $(2x + 1)^2$

3.  $(4x - 3)^2$

4.  $(-2c + 3)^2$

5.  $\left(x + \frac{1}{4}\right)^2$

6.  $\left(x - \frac{1}{5}\right)^2$

7.  $\left(\frac{1}{2}x + \frac{1}{3}\right)^2$

8.  $\left(\frac{1}{4}x - \frac{2}{3}\right)^2$

9.  $(x + 11)^2$